Kinematic Equations

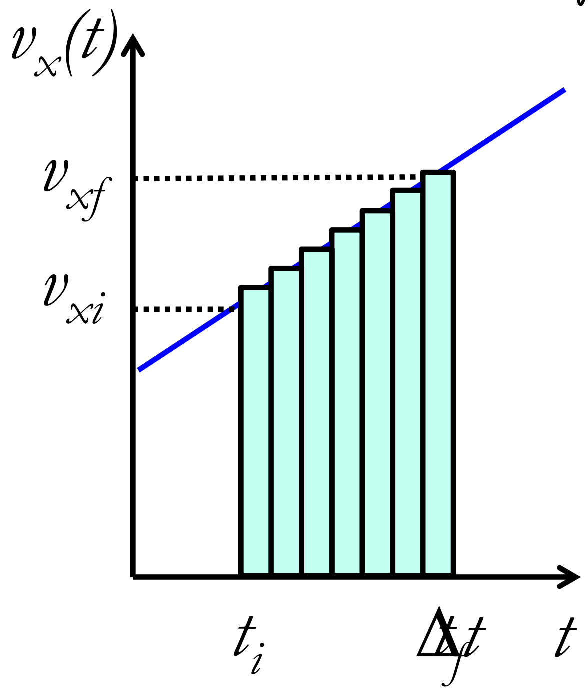
- Descriptions of Motion (words → sentences)

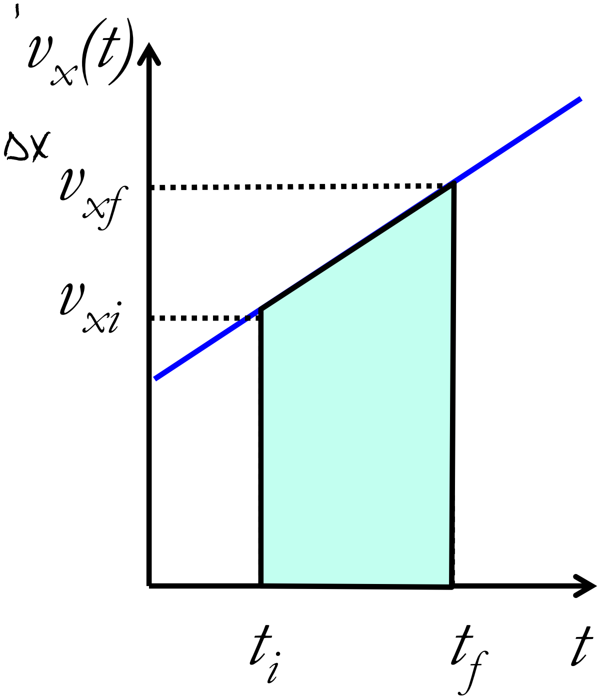
x, velocity $= \frac{\Delta x}{\Delta t}$, acceleration $a = \frac{\Delta v}{\Delta t}$

$v = v_i + at$, $a = \Delta v / \Delta t$, $a \Delta t = \Delta v = v_f - v_i$

$v = \dfrac{\Delta x}{\Delta t}$

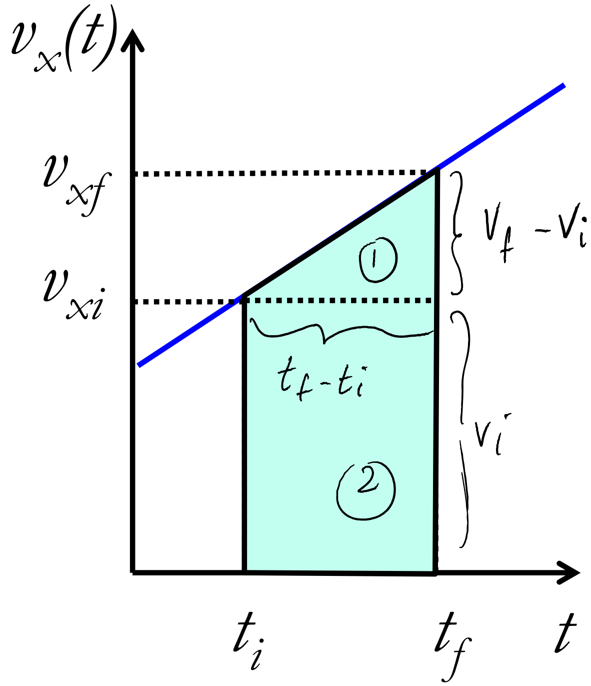
$v \Delta t = \Delta x$





$\Delta x = x_f - x_i$

total Area = total distance



$$A_1 = \frac{1}{2}(t_f - t_i)(v_f - v_i)$$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$a(t_f - t_i) = (v_f - v_i)$$

$$A_2 = v_i(t_f - t_i)$$

$$A_1 = \frac{1}{2}a(t_f - t_i)^2, \quad t_f = t, \quad t_i = 0$$

$$A_1 + A_2 = \frac{1}{2}at^2 + v_i t = x_f - x_i$$

$$\Rightarrow x = x_i + v_i t + \frac{1}{2}at^2$$

Kinematic Equations

- Descriptions of Motion (words → sentences)
- Summary:

$$v = v_i + at$$

$$x = x_i + v_i t + \frac{1}{2} a t^2$$

Helpful Hints for Kinematics

- **Time** is the key to kinematics:
  - *the* independent variable
  - horizontal axis for motion graphs
- For problem solving:
  - you can always refer everything back to the time at which it happens
  - simultaneous events occur at the same time
  - multiple objects must be referenced to the *same* coordinate system

A ball is thrown upward with an initial velocity of 20 m/s.

a) how long is the ball in the air?

b) What is the greatest height reached by the ball?

$\uparrow +$ $\quad V_{top} = 0$, $a = \text{gravity}$ $\qquad a_{grav} = g = 9.8\ m/s^2$

(a) $V = V_i + at \rightarrow$ @top $V = 0 = 20\ m/s - g t_{1/2}$, $t_{1/2} = \frac{20\ m/s}{9.8\ m/s^2}$

$= 2.04\ s$

$t_{tot} = 2 t_{1/2} = 4.08$ sec.

(b) $y = y_i^{\nearrow 0} + v_i t + \frac{1}{2} a t^2$, $y_{max} = v_i t_{1/2} - \frac{1}{2} g t_{1/2}^2 = (20)(2.04) - \frac{1}{2}(9.8)(2.04)^2$

$= 20.4\ m$

A top-fuel drag racing car can reach a speed of 100 mph in the first second of a race. (100 mph = 44.7 m/s)

(a) Find the acceleration of the car, assuming that the acceleration is constant $v_f = v_i + at$, $44.7 = 0 + a(1)$

(b) If the car continued at this acceleration, how fast would it be going at the end of the quarter-mile track? (0.25 miles is approximately 0.42 km)

$a = 44.7\ m/s^2$ → 4.6 g's

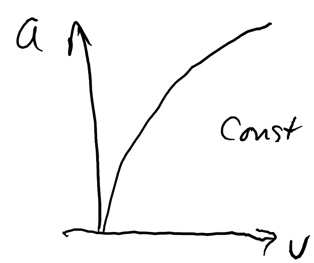
$d = x = v_i t + \frac{1}{2}at^2$ ($v_i = 0$) → solve for time → how long it takes to go 1/4 mile

$t = \sqrt{\frac{2d}{a}} = 4.33$ sec, $v_f = at = (44.7\ m/s^2)(4.33\ sec) = 193\ m/s$

430 mph

$t_{rec} = 4.455$ sec

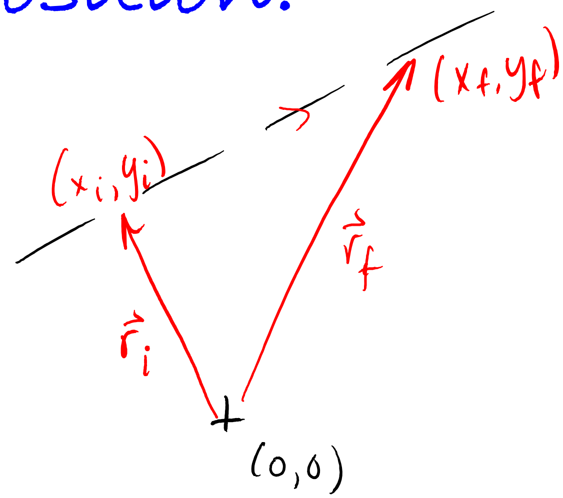
$V_{rec} = 330$ mph



Motion in 2 and 3 Dimensions

- Update: position, displacement, velocity, acceleration are *vectors* (meaning, they don’t just point in one direction)
- Problems become tractable by looking at the individual *components* of the vector equations

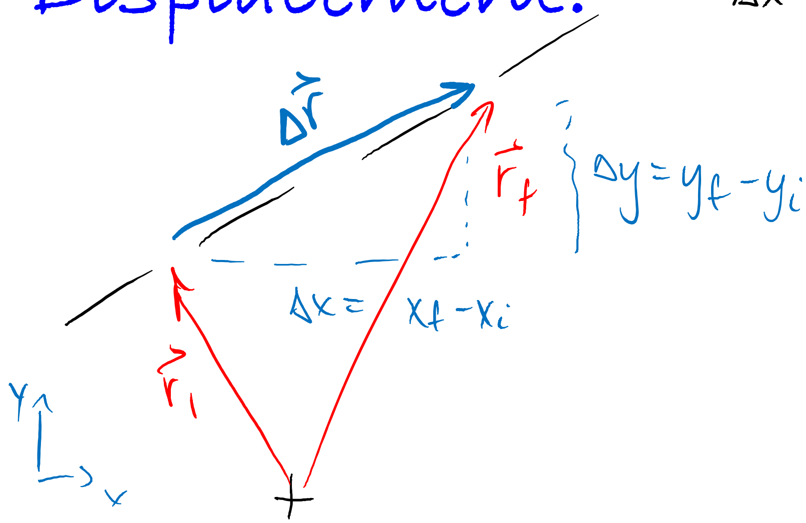
Position:



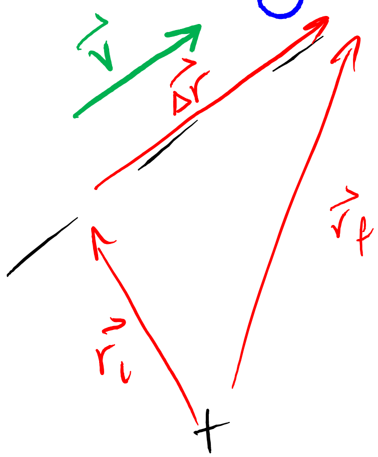
$(x, y) \rightarrow \vec{r}$

Displacement:

$\Delta x \rightarrow \Delta \vec{r}$



velocity:

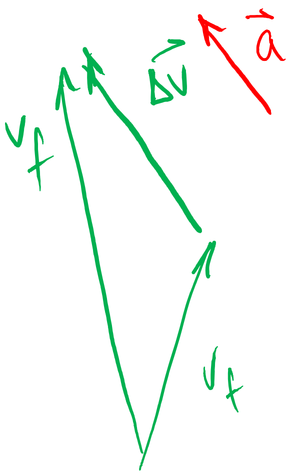


$$\vec{v} = \frac{\vec{\Delta r}}{\Delta t}$$

$$\vec{v} \parallel \vec{\Delta r}$$

$$\Delta v_x = \frac{x_f - x_i}{\Delta t}, \quad \Delta v_y = \frac{y_f - y_i}{\Delta t}$$

acceleration:



$$\vec{a} = \frac{\vec{\Delta v}}{\Delta t}$$

$$a_x = \frac{\Delta v_x}{\Delta t} \qquad a_y = \frac{\Delta v_y}{\Delta t}$$